TRAJECTORY PLANNING FOR ROBOT MANIPULATORS

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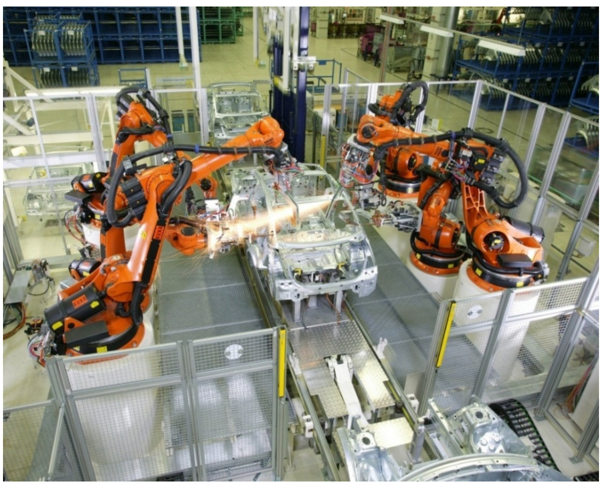
Based on MSc Research by Chinthaka Porawagama

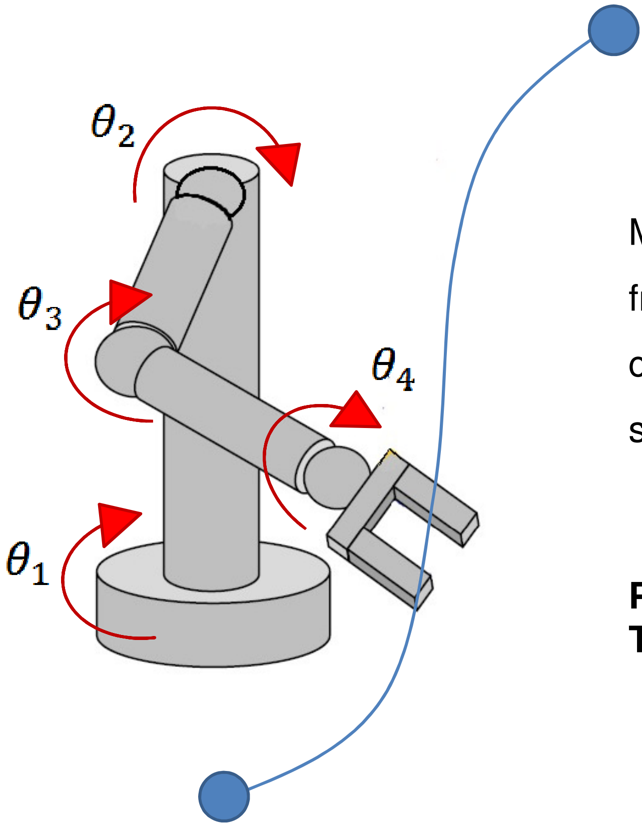
Industrial Robotics Involves in

- Pick-and-place operations
- Assembling operations
- Loading and stacking
- Automated welding, etc.

Proper motion planning is needed in these applications



Trajectory Planning

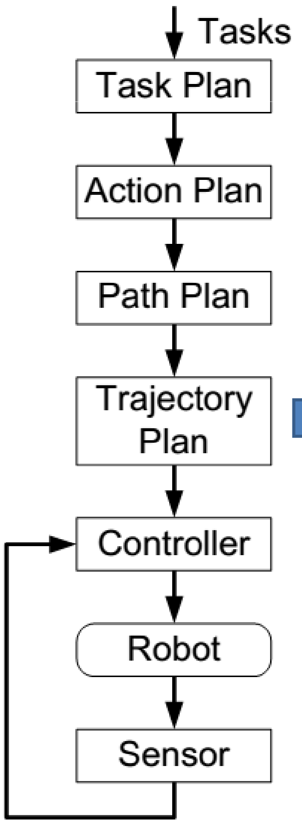


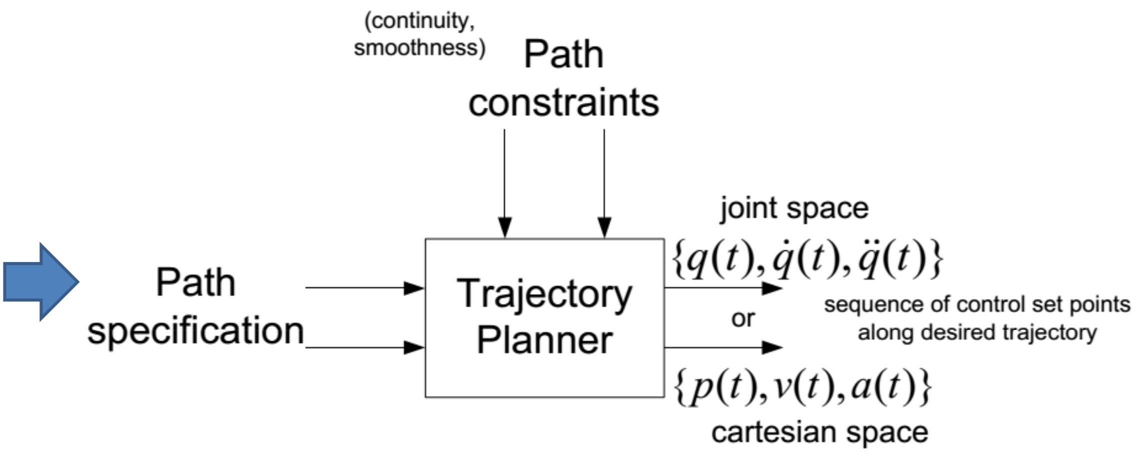
Manipulators with multi degree of freedom for accomplishing various complex manipulation in the work space

Path: only geometric description
Trajectory: timing included

Trajectory Planning

Václav Hlaváč
Czech Technical University in Prague

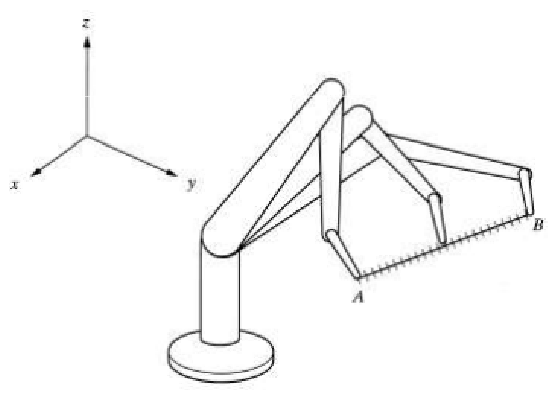




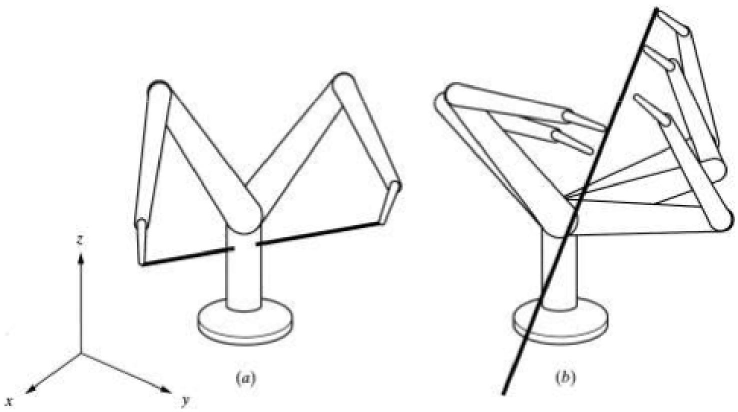
Joint Space Vs Operational Space

- Joint-space description:
  - The description of the motion to be made by the robot by its joint values.
  - The motion between the two points is unpredictable.
- Operational space description:
  - In many cases operational space = Cartesian space.
  - The motion between the two points is known at all times and controllable.
  - It is easy to visualize the trajectory, but it is difficult to ensure that singularity does not occur.

Planning in Operational Space



Sequential motions of a robot to follow a straight line.



Cartesian-space trajectory
(a) The trajectory specified in Cartesian coordinates may force the robot to run into itself, and
(b) the trajectory may requires a sudden change in the joint angles.

Planning in Operational Space

- Calculate path from the initial point to the final point.
- Assign a total time T_{path} to traverse the path.
- Discretize the points in time and space.
- Blend a continuous time function between these points
- Solve inverse kinematics at each step.

- Advantages
  - Collision free path can be obtained.
- Disadvantages
  - Computationally expensive due to inverse kinematics.
  - It is unknown how to set the total time T_{path} .

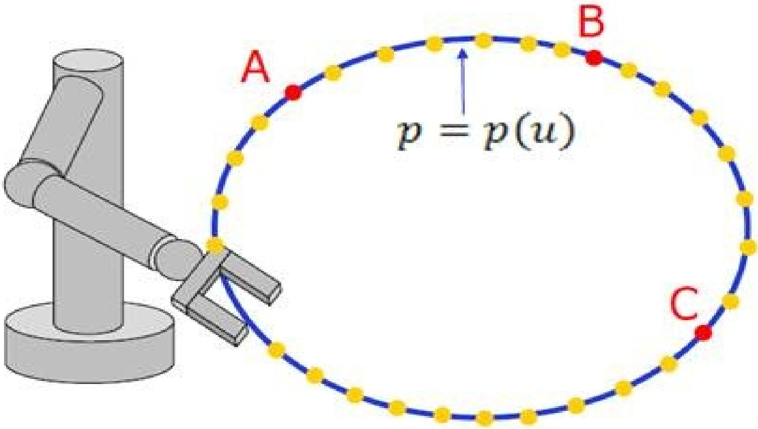
Planning in Joint Space

- Calculate inverse kinematics solution from initial point to the final point.
- Assign total time T_{path} using maximal velocities in joints.
- Discretize the individual joint trajectories in time.
- Blend a continuous function between these point.

- Advantages
  - Inverse kinematics is computed only once.
  - Can easily take into account joint angle, velocity constraints.
- Disadvantages
  - Cannot deal with operational space obstacles.

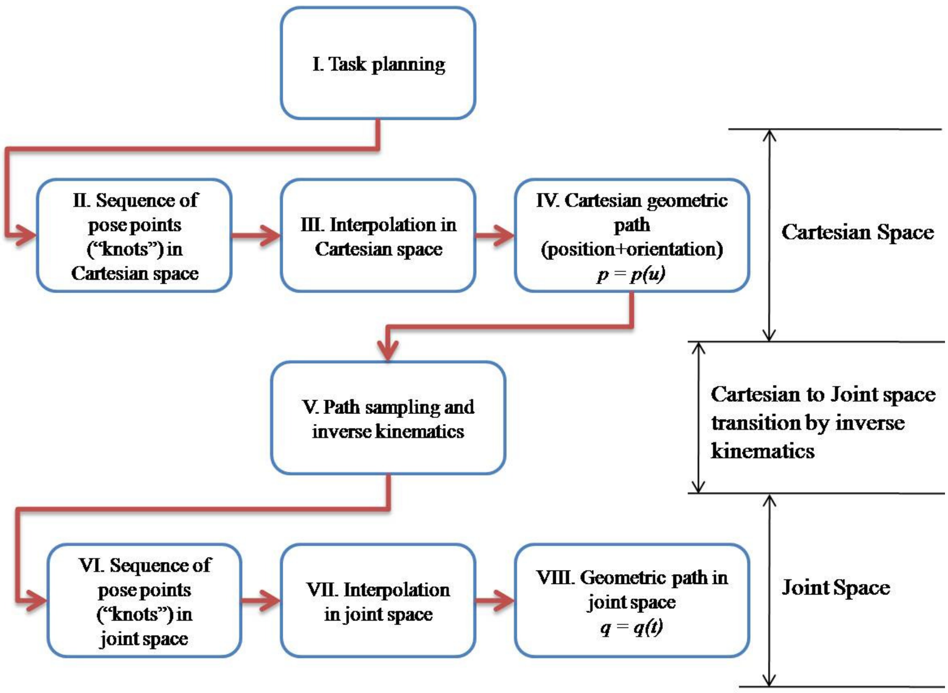
Path Definition

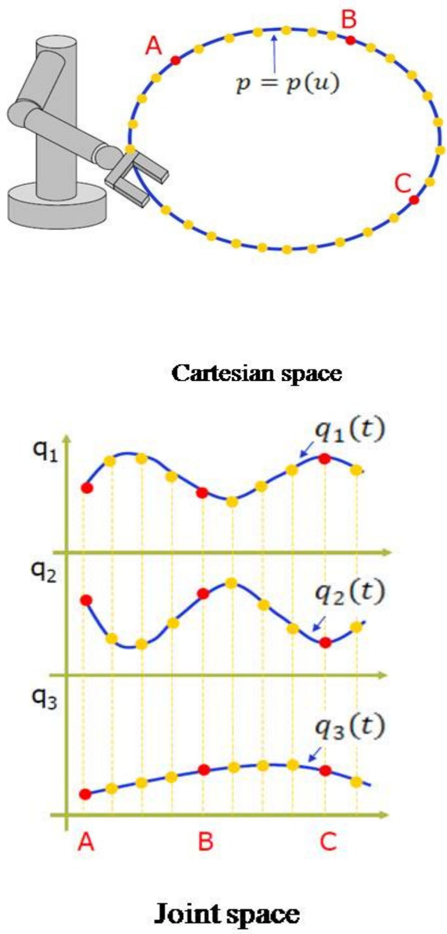
"Expressing the desired positions of a manipulator in the space, as a parametric function of time"



- B. Siciliano et al. Robotics (Modelling, planning and control), Springer, Berlin, 2009, chapter 4: Trajectory planning, pages 161-189.

Task to Trajectory

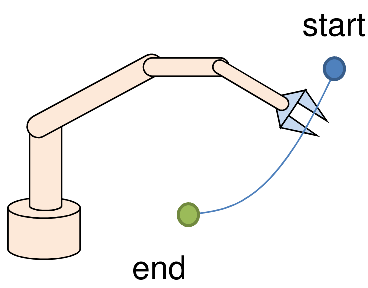




Types of Motion

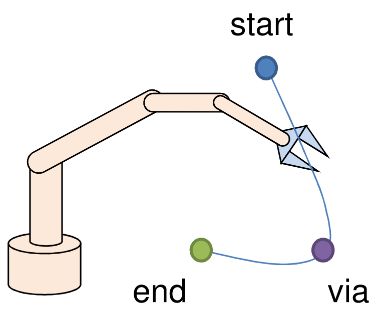
1. Point to point motion:

- End effector moves from a ***start point*** to ***end point*** in work space
- All joints' movements are coordinated for the point-to-point motion
- End effector travels in an arbitrary path



2. Motion with Via Points

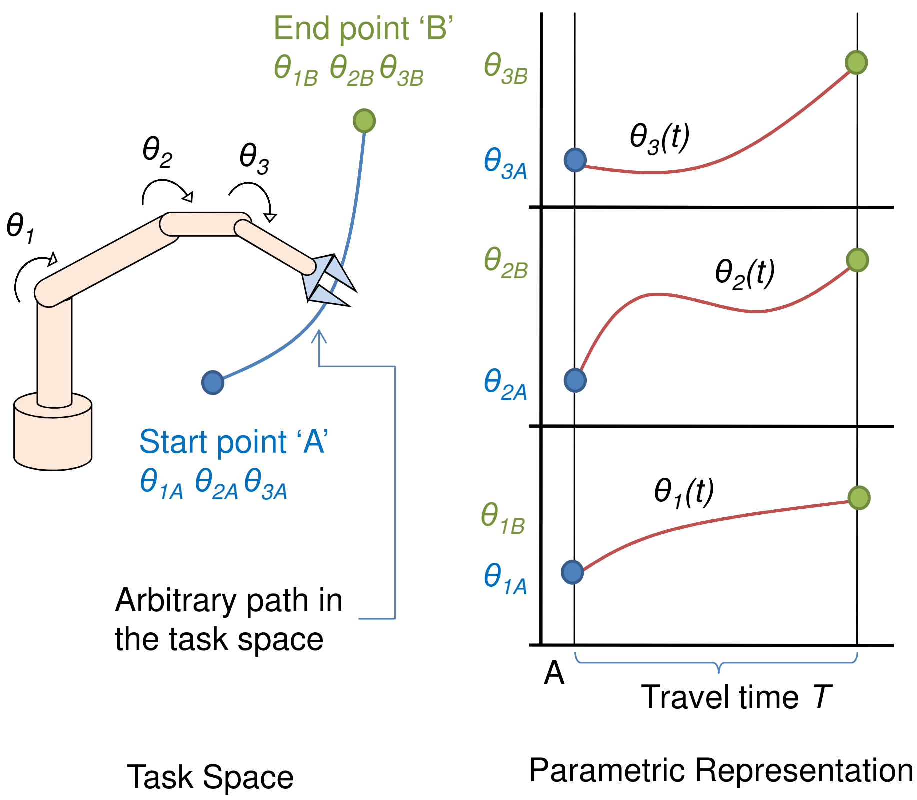
- End effector moves through an intermediate point between start and end
- End effector moves through a via point without stopping



Joint Space Planning

Point to point motion:

"Describing of joints' motions from start to end by smooth functions

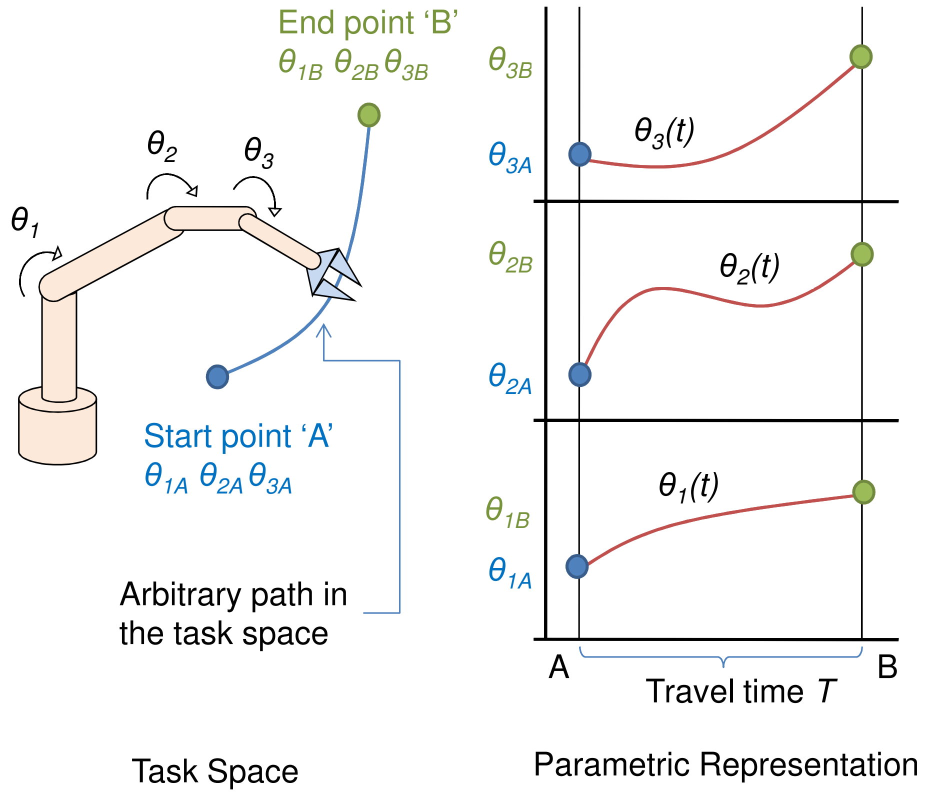


Basic stages of solving of joint space trajectory planning problem ?

Joint Space Planning

Point to point motion:

"Describing of joints' motions from start to end by smooth functions

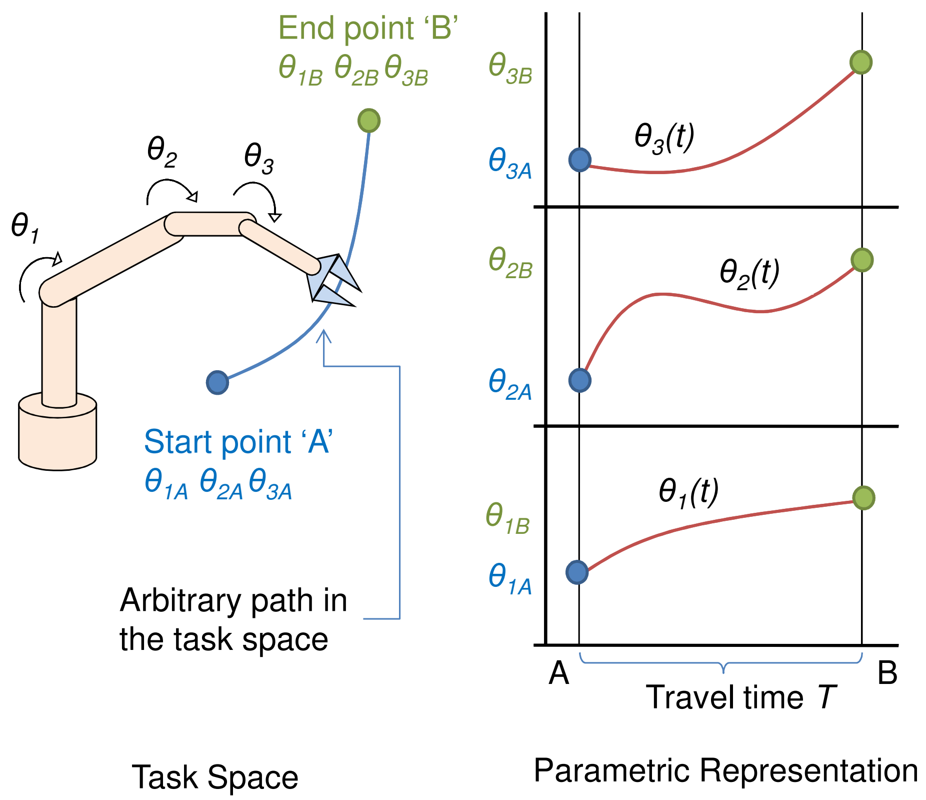


1. Inverse kinematics of start and end points (A & B)

2. Joint angles for start and end points

   $\boldsymbol{\theta_{1A}}$ $\boldsymbol{\theta_{1B}}$
   $\boldsymbol{\theta_{2A}}$ $\boldsymbol{\theta_{2B}}$
   $\boldsymbol{\theta_{3A}}$ $\boldsymbol{\theta_{3B}}$

Joint Space Planning



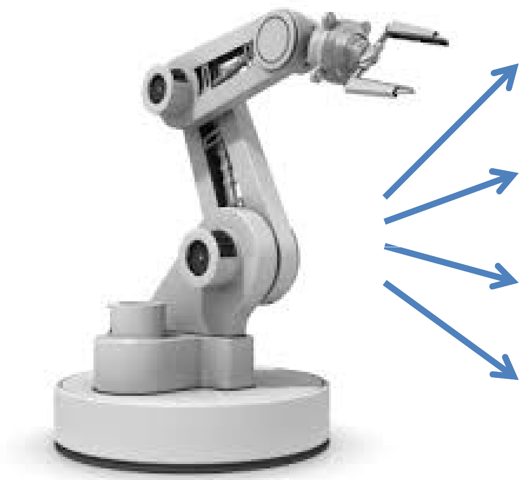
3. Interpolation of start and end joint angles by smooth functions

   $\theta_1(t)$: $\boldsymbol{\theta_{1A}} \rightarrow \boldsymbol{\theta_{1B}}$
   $\theta_2(t)$: $\boldsymbol{\theta_{2A}} \rightarrow \boldsymbol{\theta_{2B}}$
   $\theta_3(t)$: $\boldsymbol{\theta_{3A}} \rightarrow \boldsymbol{\theta_{3B}}$

4. Joint space trajectories for each joint

Smooth Motion ➡ Quality of Work

- Non smooth trajectories lead to low quality in production.



Linear Trajectory

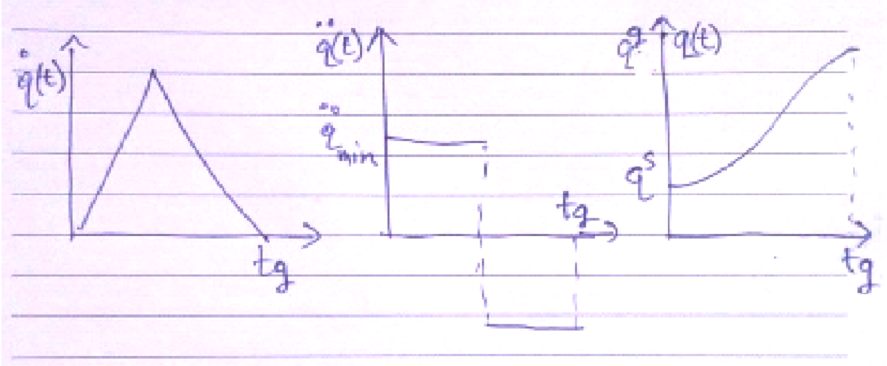
$$
\begin{aligned}
q(t) &= q^s + \frac{q^g - q^s}{t_g} t \\
\dot{q}(t) &= \frac{q^g - q^s}{t_g} \\
\ddot{q}(t) &= \begin{cases} \infty & t = 0, t_g \\ 0 & 0 < t < t_g \end{cases}
\end{aligned}
$$

- Infinite accelerations at endpoints
- Discontinuous velocity when two trajectory segments are connected (at via points)

Triangular Velocity Trajectory

$$q^g - q^s = \frac{t_g}{2}\left(\ddot{q}_{\min}\frac{t_g}{2}\right)$$

$$\ddot{q}_{\min} = \frac{4(q^g - q^s)}{t_g^2}$$



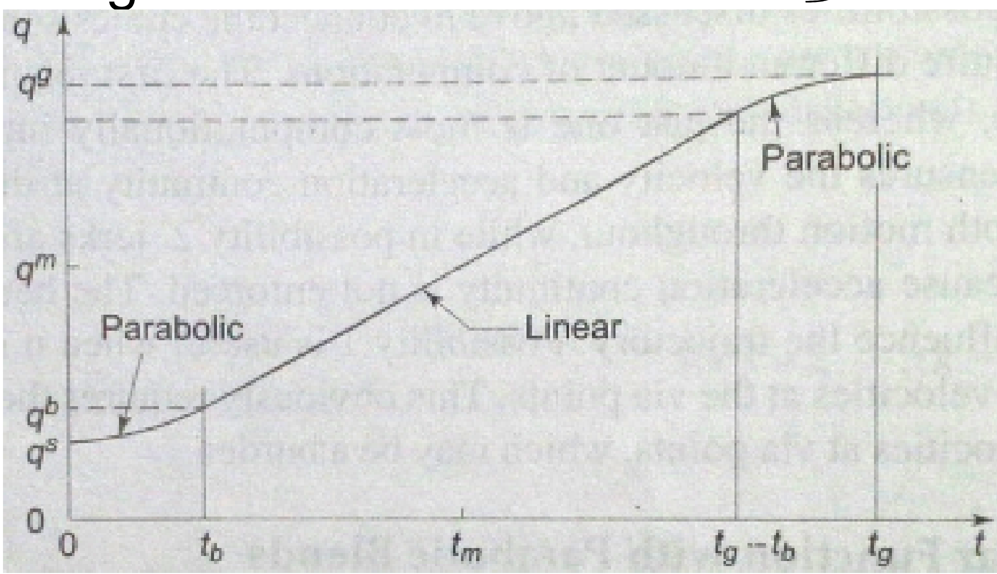
$$q(t) = \begin{cases} q^s + 0.5\ddot{q}_{\min}t^2 & 0 \le t < 0.5t_g \\ 0.5(q^s + q^g) - 0.5\ddot{q}_{\min}t^2 & 0.5t_g \le t \le t_g \end{cases}$$

- Acceleration discontinuity at endpoints and at the midpoint of the trajectory

Linear Trajectory with Parabolic Blends

- Zero acceleration in middle segments.
- Constant acceleration at end segments.

Acceleration discontinuity at blend points



Linear Trajectory with Parabolic Blends

- Total angular motion

$$S = 2(parabolic) + Linear$$

$$q^g - q^s = 2\times\frac{1}{2}\ddot{q}^b t_b^2 + \dot{q}^b (t_g - t_b)$$

$$\Downarrow$$

$$\ddot{q}^b t_b^2 - \ddot{q}^b t_g t_b + (q^g - q^s) = 0$$

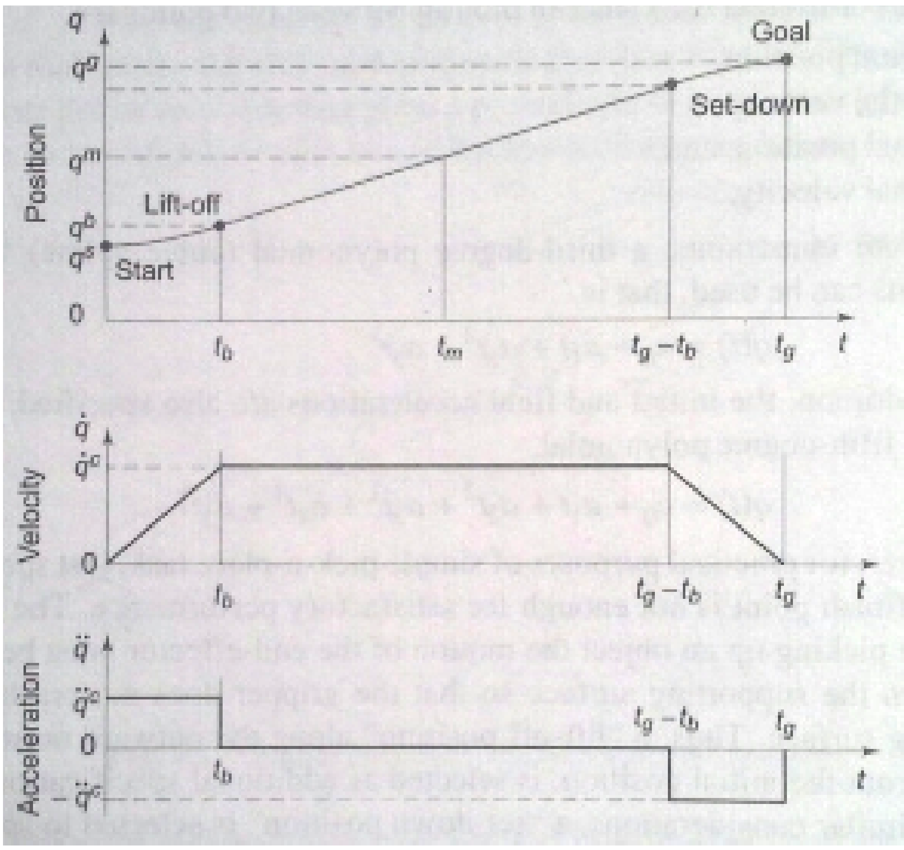
$$\Downarrow$$

$$t_b = \frac{t_g}{2} \pm \frac{1}{2}\sqrt{t_g^2 - \frac{4(q^g - q^s)}{\ddot{q}^b}}$$

- For a linear part to exist

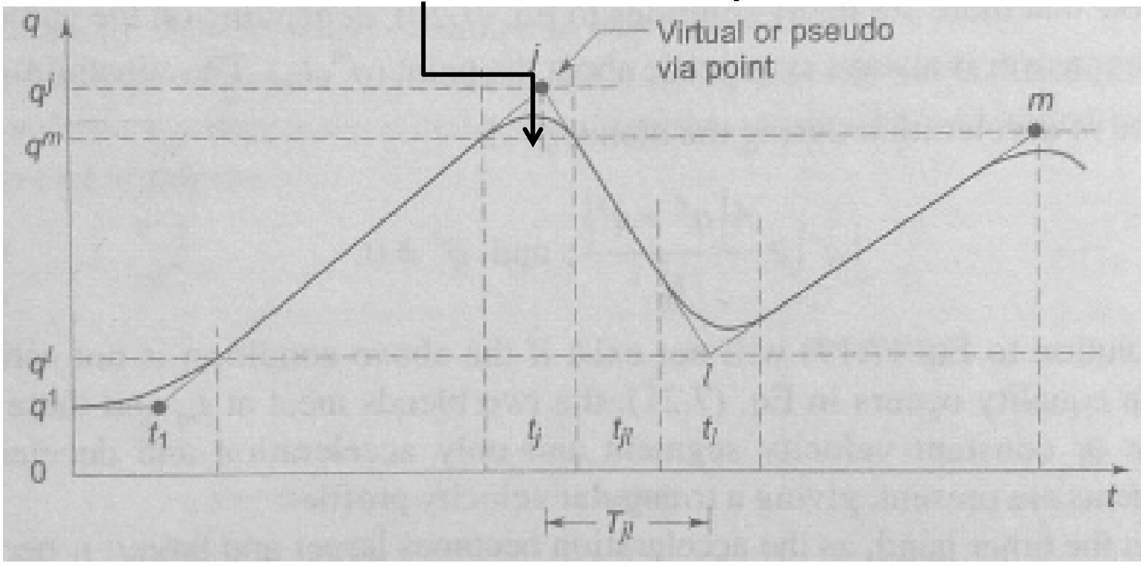
$$t_g^2 - \frac{4(q^g - q^s)}{\ddot{q}^b} > 0 \;\Rightarrow\; \ddot{q}^b > \frac{4(q^g - q^s)}{t_g^2}$$ ← Minimum joint acceleration

Linear Trajectory with Parabolic Blends



Inclusion of Via Points into a Linear Trajectory with Parabolic Blends

- Via points (knot points) can be introduced between start and goal (q^s, q^g) positions with constant acceleration at via point.



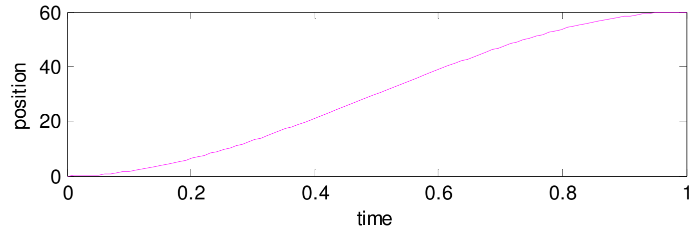
Multi-stage linear parabolic blend spline

Cubic Polynomial (Bring in Smoothness)

Joint Position

Cubic

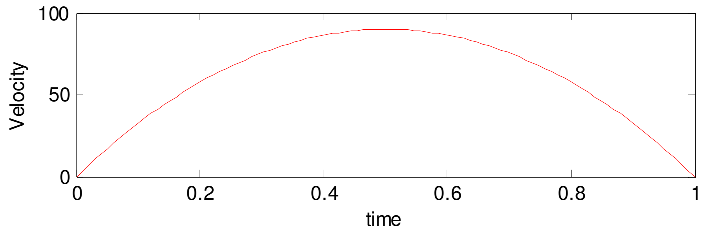
$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (1)$$



Joint Velocity

Parabolic

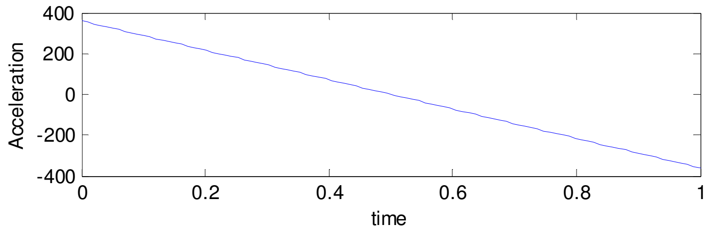
$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 \quad (2)$$



Joint Acceleration

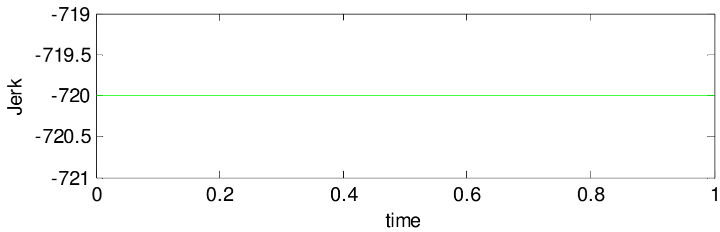
Linear

$$\ddot{q}(t) = 2a_2 + 6a_3 t \quad (3)$$



Joint Jerk

$$\dddot{q}(t) = 6a_3 \quad (4)$$



Cubic Polynomial (zero sped at end-points)

- Satisfies position and velocity at end-points
- Eg: zero speed at end-points

$$
\left.\begin{array}{rcl}
q(0) & = & q^s \\
q(t_g) & = & q^g \\
\dot{q}(0) & = & 0 \\
\dot{q}(t_g) & = & 0
\end{array}\right\} \Rightarrow
\begin{array}{ll}
(1)\ \text{for } t=0, & q^s = a_0 \\
(1)\ \text{for } t=t_g, & q^g = a_0 + a_1 t_g + a_2 t_g^2 + a_3 t_g^3 \\
(2)\ \text{for } t=0, & \dot{q}^s = a_1 \\
(2)\ \text{for } t=t_g, & \dot{q}^g = a_1 + 2a_2 t_g + 3a_2 t_g^2
\end{array}
$$

$$
\begin{array}{rcl}
a_0 & = & q^s \\
a_1 & = & \dot{q}^s (=0) \\
a_2 & = & \dfrac{3(q^g - q^s)}{t_g^2} \\
a_3 & = & \dfrac{-2(q^g - q^s)}{t_g^3}
\end{array}
$$

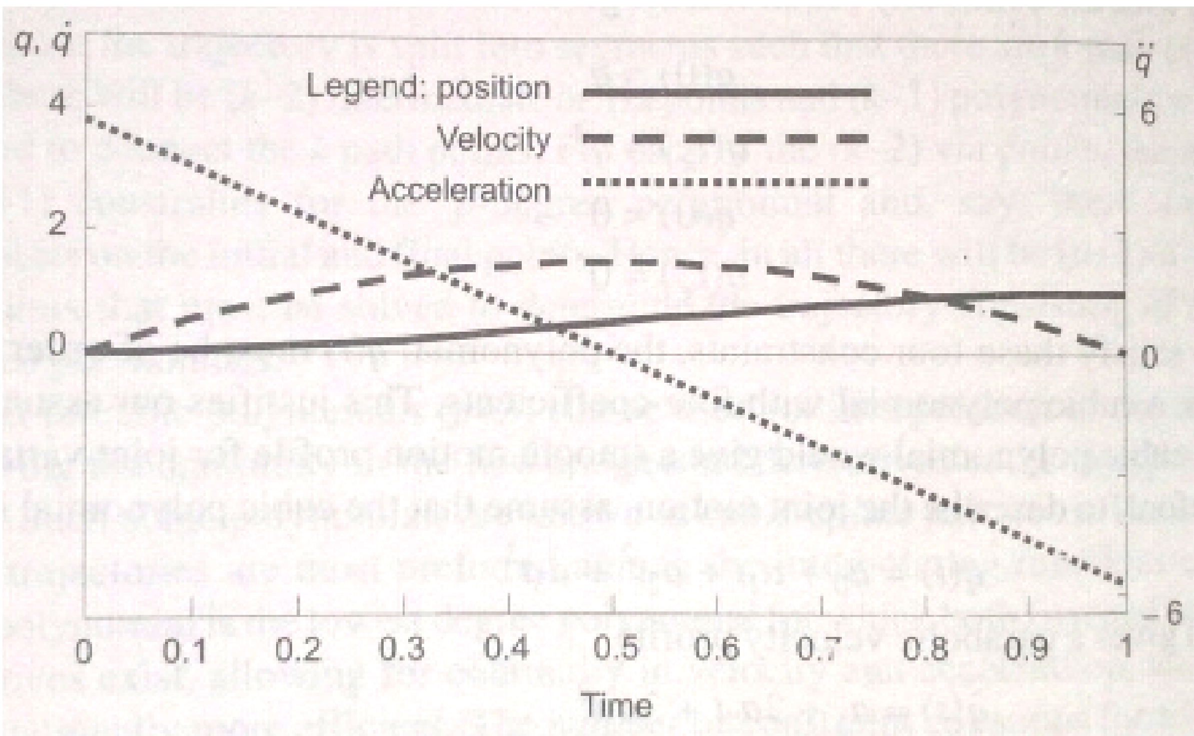
$$
q(t) = q^s + \frac{3}{t_g^2}(q^g - q^s)t^2 - \frac{2}{t_g^3}(q^g - q^s)t^3
$$

$$
0 \le t \le t_g
$$

- Acceleration is linear and uncontrollable

$$
\ddot{q}(t) = 2a_2 + 6a_3 t \quad 0 \le t \le t_g
$$

Cubic Polynomial (zero sped at end-points)



Cubic Polynomial (nonzero speeds at end- points)

- Satisfies position and velocity at end-points
- Eg: non-zero speeds at end-points

$$\left.\begin{aligned} q(0) &= q^s \\ q(t_g) &= q^g \\ \dot{q}(0) &= \dot{q}^s \\ \dot{q}(t_g) &= \dot{q}^g \end{aligned}\right\} \Rightarrow \begin{aligned} &(1)\text{ for } t=0, && q^s = a_0 \\ &(1)\text{ for } t=t_g, && q^g = a_0 + a_1 t_g + a_2 t_g^2 + a_3 t_g^3 \\ &(2)\text{ for } t=0, && \dot{q}^s = a_1 \\ &(2)\text{ for } t=t_g, && \dot{q}^g = a_1 + 2a_2 t_g + 3a_2 t_g^2 \end{aligned}$$

$$\begin{aligned} a_0 &= q^s \\ a_1 &= \dot{q}^s \\ a_2 &= \frac{3}{t_g^2}(q^g - q^s) - \frac{1}{t_g}(\dot{q}^g - \dot{q}^s) \\ a_3 &= -\frac{2}{t_g^3}(q^g - q^s) + \frac{1}{t_g^2}(\dot{q}^g - \dot{q}^s) \end{aligned}$$

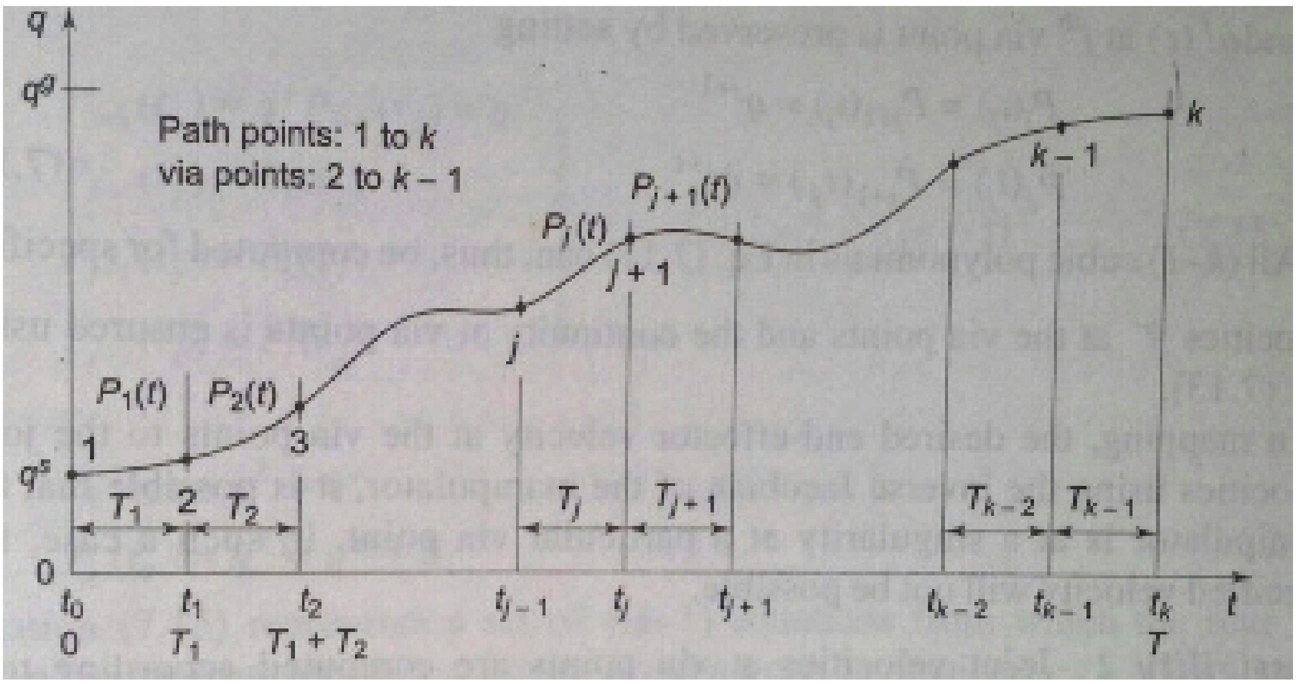
$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \Leftarrow$$
$$0 \le t \le t_g$$

- Acceleration is linear and uncontrollable

$$\ddot{q}(t) = 2a_2 + 6a_2 t \quad 0 \le t \le t_g$$

Cubic Spline Trajectory

- Stitching cubic polynomials together



- Acceleration is not continuous at via (stitching points)

5^{th} Order Polynomial (more oscillatory)

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

$$\dddot{q}(t) = 6a_3 + 24a_4 t + 60a_5 t^2$$